

# Theory input for the measurement of $W+c$ production at the LHC

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High Energy Physics Seminar  
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# Outline:

→ Predictions for  $W+c!$  ...

→ ... and why you want to compute them

- *NNLO QCD predictions for  $W+c$ -jet production at the LHC*

[Czakon, Mitov, MP, Poncelet; 2011.01011]

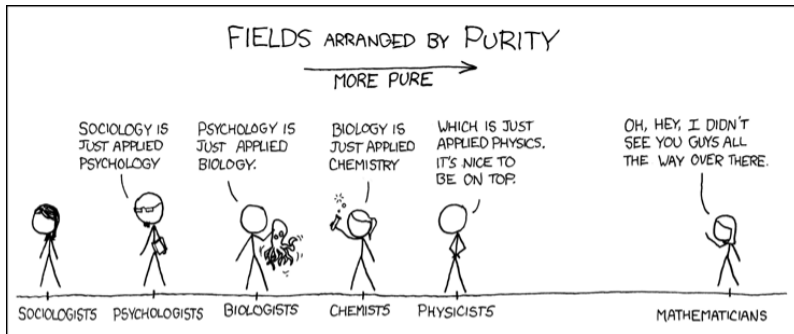
- *A detailed investigation of  $W+c$ -jet at the LHC*

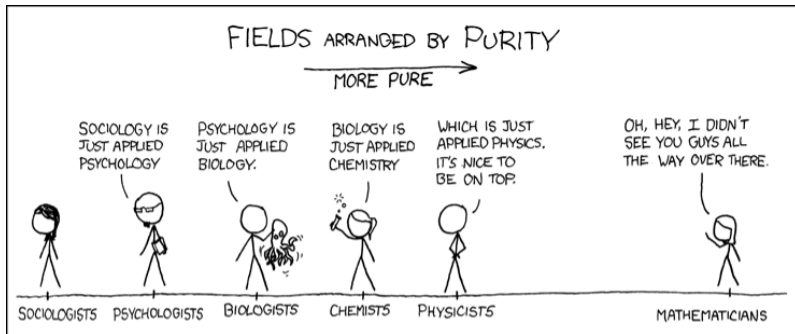
[Czakon, Mitov, MP, Poncelet; 2212.00467]

- **Comparison to data!**

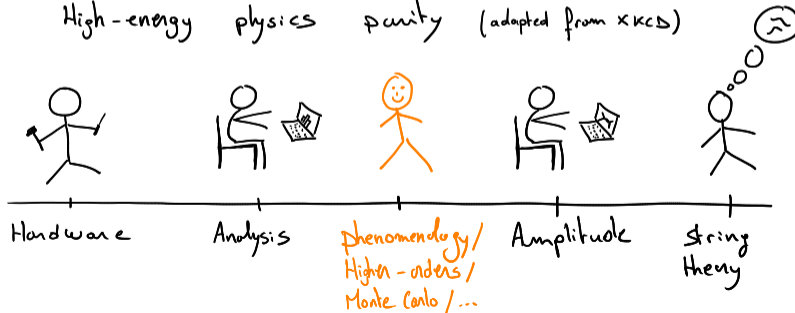
Measurement of the production cross section for a  $W$  boson in association with a charm quark in proton-proton collisions at  $\sqrt{s} = 13$  TeV

[CMS + Czakon, Mitov, MP, Poncelet; 2308.02285]





High-energy physics purity (adapted from XKCD)

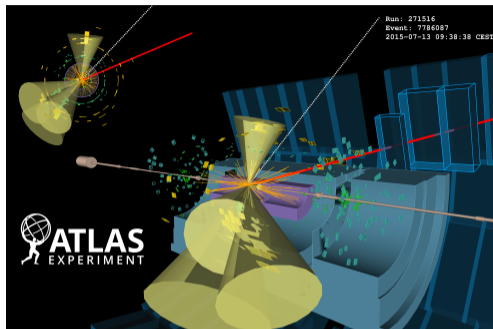




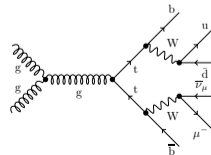
→ Illustration of Giordano Bruno's philosophical ideas (XVI<sup>th</sup> century)

LHC: Great tool to probe fundamental interactions at high energies

→ Cross talk between **experiment** and **theory**



$$pp \rightarrow t^* \bar{t}^* \rightarrow (W^* \rightarrow \nu_\mu \mu^-) (W^* \rightarrow jj) b \bar{b}$$



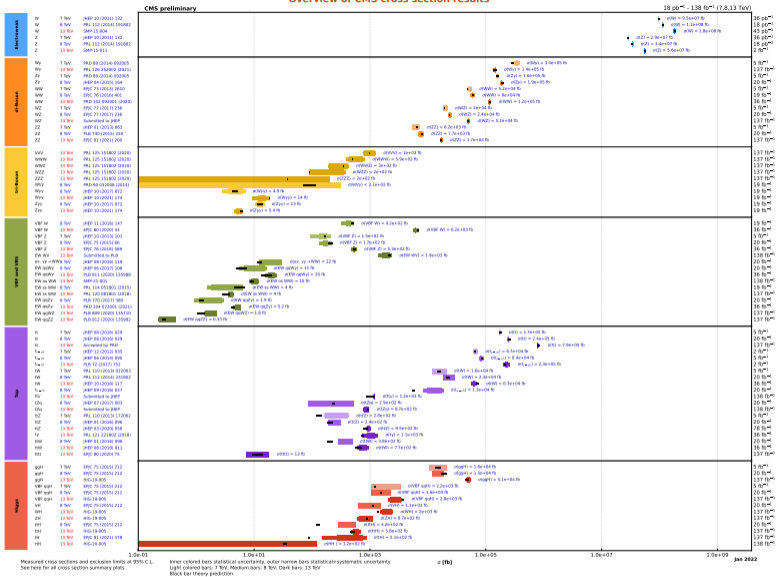
- Greatest achievement of the LHC so far:

## Discovery of the Higgs boson



→ Great interest in measuring properties of the Higgs boson ...  
... but there are also other interesting things

# Overview of CMS cross section results



→ Cross-sections measurements machine!



- NLO QCD:

[Giele et al.; hep-ph/9302225], [Arnold et al.; Nucl.Phys. B319 (1989) 37-71, Phys.Rev. D40 (1989) 912], [Campbell et al.; hep-ph/0202176, 0809.3003, 1107.3714]

- NNLO QCD:

[Boughezal et al.; 1504.0213, 1602.06965], [Gehrmann-De Ridder et al.; 1901.11041], [MP et al.; 2204.12394]

- NLO EW:

[Kühn et al.; hep-ph/0703283, 0708.0476], [Hollik et al.; 0707.2553], [Denner et al.; 0906.1656]

- Combinations of QCD and EW corrections:

[Kallweit et al.; 1412.5157, 1511.08692], [Lindert et al.; 1511.08692], [Biederman, MP et al.; 1704.05783]

- NLO QCD for polarised W-jet:

[Bern et al.; 1103.5445] [Stirling, Vryonidou; 1204.6427]

- NNLO QCD for polarised W-jet:

[MP, Popescu, Poncelet; 2109.14336]

- NLO QCD for  $W+c$ -jet:

[Giele, Keller, Laenen; hep-ph/9511449] [Stirling, Vryonidou; 1203.6781]

- NLO QCD for  $W+c$ -jet:

[Czakon, Mitov, MP, Poncelet; 2011.01011], [Gehrmann-De Ridder et al.; 2311.14991]

- NNLO QCD+NLO EW for  $W+c$ -jet:

[Czakon, Mitov, MP, Poncelet; 2212.00467]

- NLO QCD+PS for  $W+c$ -jet:

[Bevilacqua, Garzelli, Kardos, Toth; 2106.11261], [Ferrario Ravasio, Oleari; 2304.13791]

- Study of charm production in context of strange PDF:

[Lai et al.; hep-ph/0702268], [Yalkun, Dulat; 1908.00026], [Faura et al.; 2009.00014]

- NNLO QCD  $Z+b$ -jet - [Gauld et al.; 2005.03016], NNLO QCD  $W+bb$  - [Hartanto et al.; 2209.03280]

→ This work: **NNLO QCD (+ NLO EW) computation for  $W+c$ -jet production**



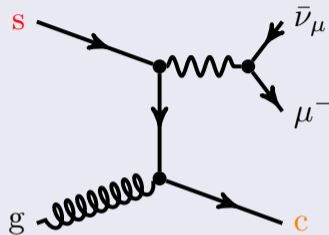
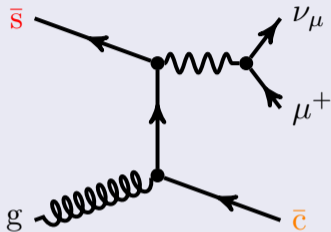
- Private Monte Carlo STRIPPER // MoCANLO  
[Czakov, Heymes, Poncelet; 1005.0274, 1101.0642, 1408.2500] // [Denner, Feger, Lombardi, MP, Pelliccioli, Schmidt, Schwan]
- Tree level: AVH [Bury, van Hameren; 1503.08612]
- One-loop: OPENLOOP2 [Buccioni et al.; 1907.13071] // RECOLA [Actis et al.; 1211.6316, 1605.01090]
- Two-loop: [Gehrmann, Tancredi; 1112.1531]  
→ using GINAC [Bauer, Frink, Kreckel], [Vollinga, Weinzierl; hep-ph/0410259]
- Complex-mass scheme [Denner et al.; hep-ph/9904472, hep-ph/0505042, hep-ph/0605312]
- PDF: LHAPDF [Buckley et al.; 1412.7420]

# PART I

- *NNLO QCD predictions for  $W+c$ -jet production at the LHC*

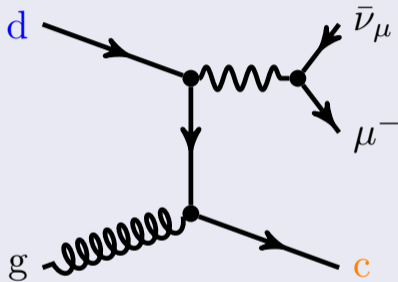
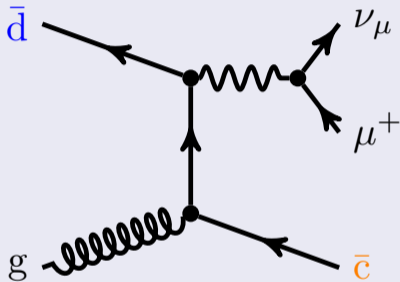
[Czakon, Mitov, MP, Poncelet; 2011.01011]

# LO process (1)



- Direct link between  $W+c$  measurements and strange PDF  
→ main motivation to be interested in this process
- Test of (perturbative) QCD  
→  $s$ - $\bar{s}$  asymmetry predicted at 3-loop in QCD [Catani, de Florian, Rodrigo, Vogelsang; hep-ph/0404240]
- Study of flavour jets
- Further test of the Standard Model

## LO process (2)



- With non-diagonal CKM matrix ( $V_{cd} \neq 0$ ) ...  
...more complicated interpretation in terms of strange PDF

$$R_{W^{\pm}j_c} = \frac{\sigma_{W^+j_c}}{\sigma_{W^-j_c}} \sim (|V_{cs}|^2\bar{s} + |V_{cd}|^2\bar{d}) / (|V_{cs}|^2s + |V_{cd}|^2d)$$

PDF set	$V_{cd}$	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^{\pm}j_c}$
NNPDF31 LO	= 0	9.8395(4)	10.4654(4)	0.94020(5)
	≠ 0	12.0725(4)	14.2624(5)	0.84646(4)
NNPDF31 NLO	= 0	22.593(2)	23.718(2)	0.95260(6)
	≠ 0	24.500(9)	27.29(1)	0.8977(5)
CT18 NLO	= 0	21.675(2)	21.675(2)	1.0000(1)
	≠ 0	23.477(9)	25.252(8)	0.9297(5)

- Large differences between PDF sets ...

# Inclusion of higher orders

$$pp \rightarrow W^+ j_c$$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	✓	✓	✓
$sg$	X	X	✓
$s\bar{s}$	X	✓	✓
$\bar{s}\bar{s}$	X	✓	✓
$\bar{s}q$	X	✓	✓
$qq'$	X	✓	✓
$gq$	X	X	✓
$gg$	X	✓	✓

$$pp \rightarrow W^- j_c$$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	X	X	✓
$sg$	✓	✓	✓
$s\bar{s}$	X	✓	✓
$ss$	X	✓	✓
$sq$	X	✓	✓
$qq'$	X	✓	✓
$gq$	X	X	✓
$gg$	X	✓	✓

- Higher-order corrections further complicates the picture
- Interpretation of  $W+c$ -jet is not trivial



- Event selection

$$\begin{aligned} p_{T,\ell} &> 20 \text{ GeV}, & |\eta_\ell| &< 2.5 \\ p_{T,\text{miss}} &> 25 \text{ GeV}, & m_T^W &> 40 \text{ GeV}. \end{aligned}$$

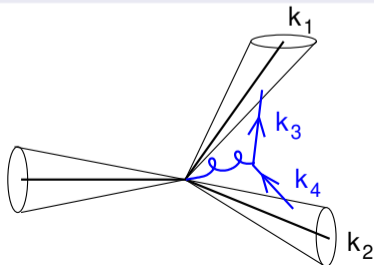
One and only one flavoured c-jet with:

$$p_{T,j_c} > 25 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

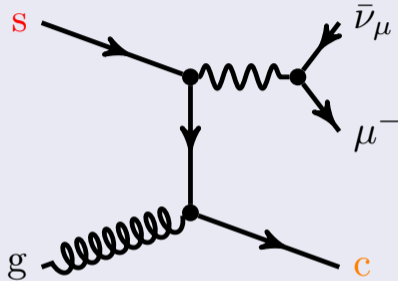
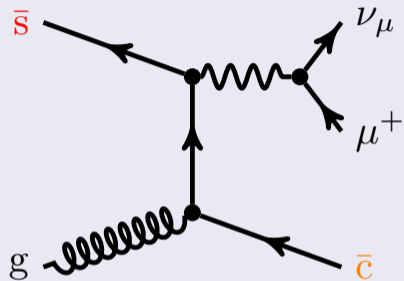
- NNPDF31 sets with  $\alpha_s = 0.118$  [Ball et al.; 1706.00428]
- $\mu = \frac{1}{2} (E_{T,W} + p_{T,j_c})$  where  $E_{T,W} = \sqrt{M_W^2 + (\vec{p}_{T,\ell} + \vec{p}_{T,\nu})^2}$
- **flavour  $k_T$  algorithm with  $R = 0.4$**  [Banfi, Salam, Zanderighi; hep-ph/0601139]

# Jet algorithm

- Beyond NLO, flavour jet algorithm is required
  - Otherwise no IR-safe definition of flavour jets
  - Soft wide angle radiations are problematic
- flavour  $k_T$  algorithm with  $R = 0.4$  [Banfi, Salam, Zanderighi; hep-ph/0601139]
  - Soft radiations are clustered first
  - rules:
    - $c + c = j$  or  $c + \bar{c} = j$
    - $c + c + \bar{c} = j_c$  or  $\bar{c} + c + \bar{c} = j_c$

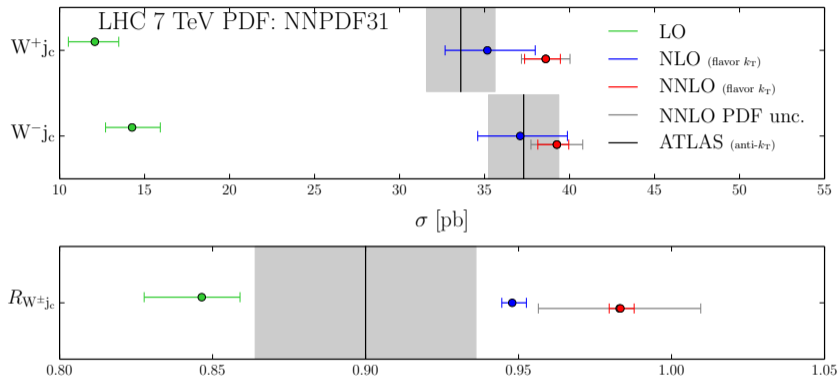


# Features of the computation



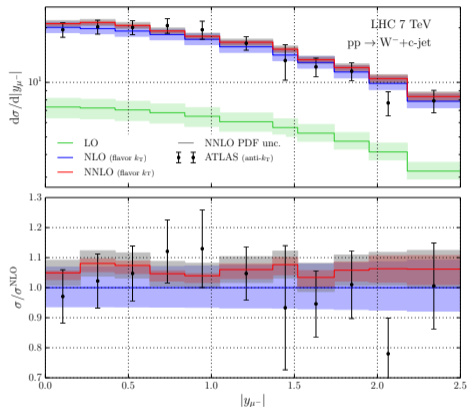
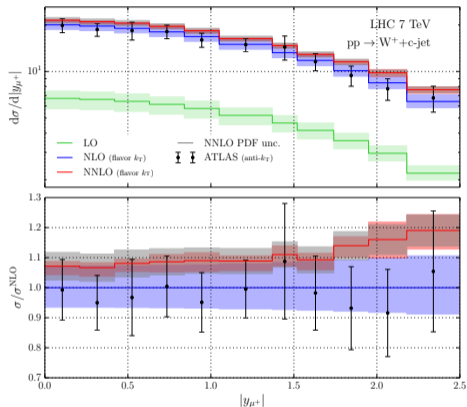
- NNLO QCD computation to  $pp \rightarrow \mu^+ \nu_\mu j_c$  and  $pp \rightarrow \mu^- \bar{\nu}_\mu j_c$
- 5-flavour scheme
- PDF uncertainty computed at NNLO using [Carrazza et al.; 1602.00005]
- $V_{cd} \neq 0$  at LO when comparing against data

# Th. vs. Exp. - cross section



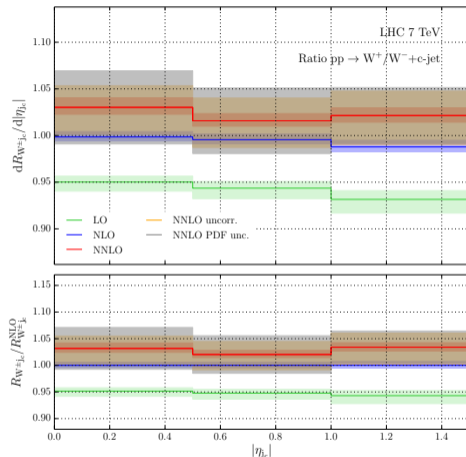
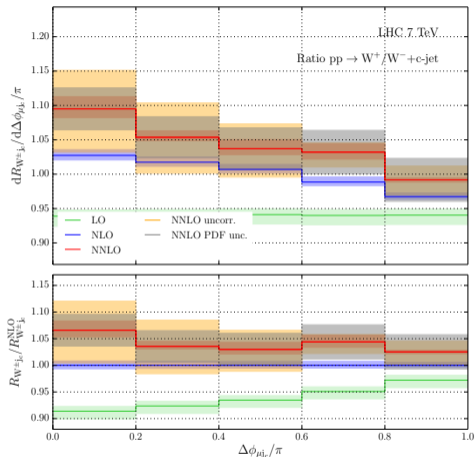
- PDF uncertainty dominant over NNLO scale uncertainty
- NNLO QCD prediction tends to be larger for the + signature
  - Not statistically relevant [ATLAS; 1402.6263]

# Th. vs. Exp. - Differential distribution



Similar picture as for the total cross section  
→ General good agreement

# Differential distributions - ratio



As for total cross section, PDF uncertainty are dominant in ratios  
→ Uncorrelated scale uncertainty more conservative

- Difference in the jet algorithms (flavoured  $k_T$  vs. anti- $k_T$ )
  - Estimated to be 12% in  $Z + b$  [Gauld et al.; 2005.03016] ...
  - ... but difficult to translate to  $W + c$

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- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element  
~ few per cent



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- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element  $\sim$  few per cent
- Absence of EW corrections  $\sim$  - few per cent

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~ few per cent
- Absence of EW corrections ~ - few per cent
- PDF uncertainty

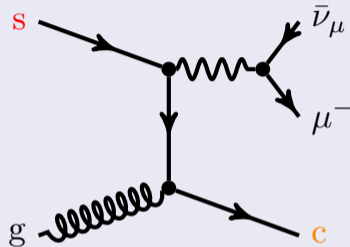
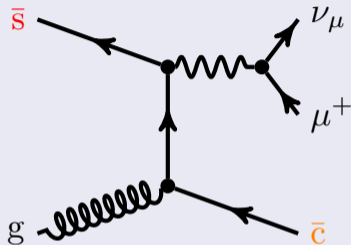
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- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element  
~ few per cent
- Absence of EW corrections ~ - few per cent
- PDF uncertainty
- Definition of the experimental measurement ?
  - D meson and not charm jet ?

# PART II

- *A detailed investigation of  $W+c$ -jet at the LHC*

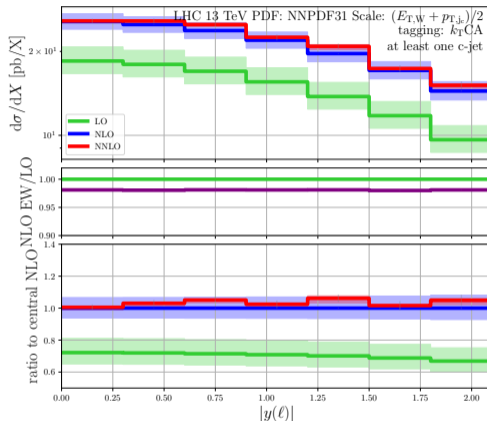
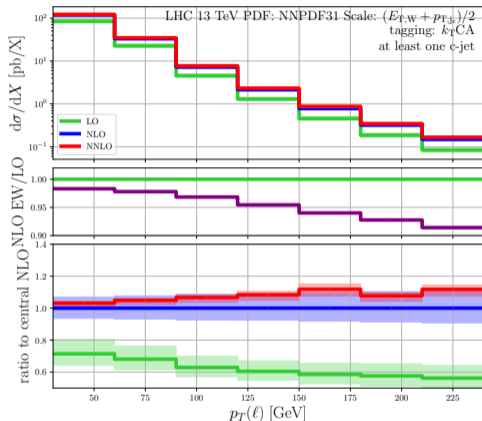
[Czakon, Mitov, MP, Poncelet; 2212.00467]

# Feature of the (new) computation



- Full CKM dependence up to NNLO QCD
- NLO EW
- Study of flavour-jet algorithm
- Study of experimental definition
- 13 TeV setup

# Best predictions @ 13 TeV - Differential distributions



- Good perturbative behaviour for QCD corrections
- Sudakov logarithm for EW corrections

# Best predictions @ 13 TeV - cross sections

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
LO	$113.817(2)^{+12.4\%}_{-9.87\%}$	$119.711(2)^{+12.4\%}_{-9.88\%}$	$0.95076(2)^{+0.013\%}_{-0.021\%}$
NLO	$162.4(1)^{+7.2\%}_{-6.6\%}$	$168.1(1)^{+6.9\%}_{-6.4\%}$	$0.9659(9)^{+0.29\%}_{-0.21\%}$
NNLO	$168.6(8)^{+0.7\% +3.8\%(PDF)}_{-2.1\% -3.8\%(PDF)}$	$173.9(1.9)^{+0.6\% +3.7\%(PDF)}_{-1.8\% -3.7\%(PDF)}$	$0.96(1)^{+0.2\% +2.1\%(PDF)}_{-0.3\% -2.1\%(PDF)}$

- PDF uncertainty dominant at NNLO QCD

# Best predictions @ 13 TeV - cross sections

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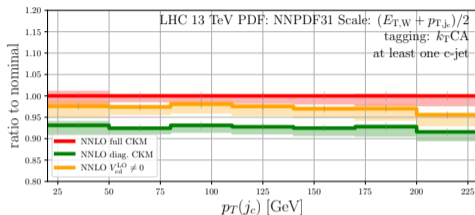
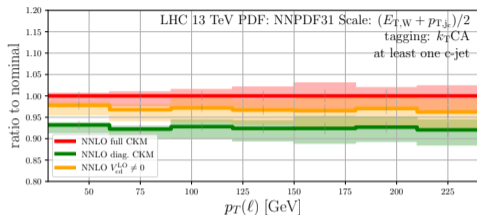
- PDF uncertainty dominant at NNLO QCD

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
NLO EW	117.399(2)	111.627(2)	0.95084(2)
$\delta_{\text{NLO EW}}[\%]$	-1.93	-1.92	-0.01

- EW corrections null in the ratio

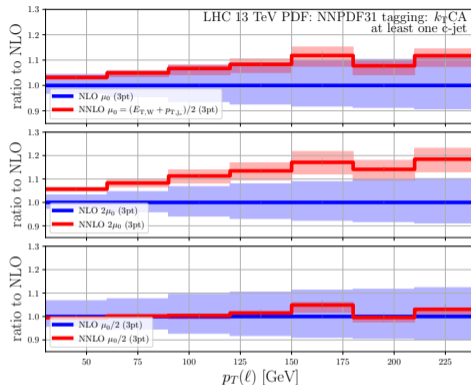
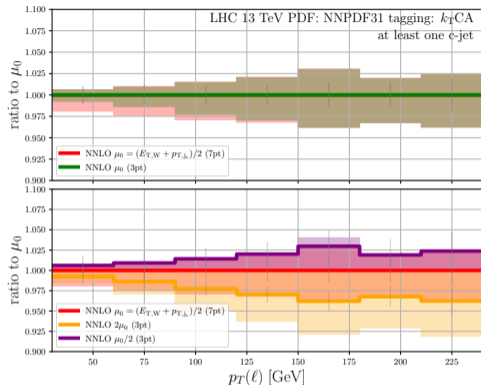


# Effect of non-diagonal CKM @ NNLO QCD



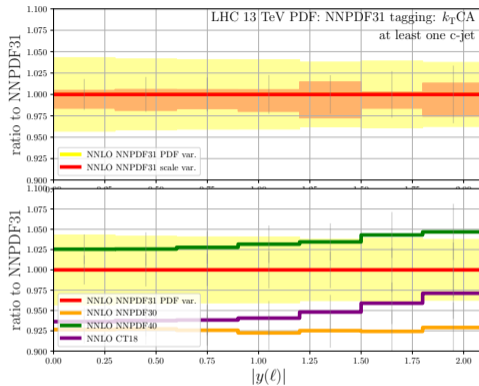
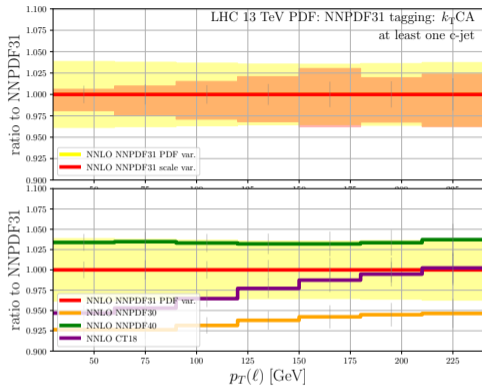
- full CKM / no CKM  $\sim 7.5\text{--}11\%$
- full CKM /  $V_{cd}^{LO} \neq 0 \sim 3\%$ 
  - Original approximation rather good
  - Full CKM dependence up to NNLO QCD for precise predictions

# Scale setting



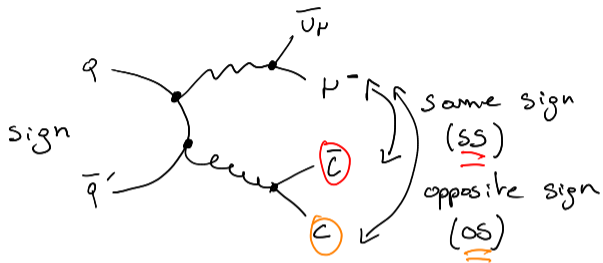
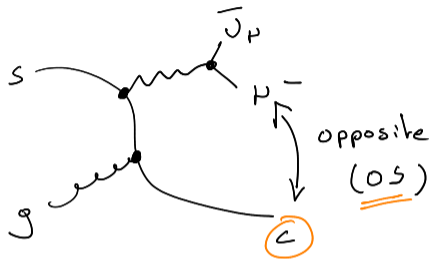
- $\mu_0 = \frac{1}{2} (E_{T,W} + p_{T,j_c})$
- For  $p_{T,\ell}$ ,  $\mu_0/2$  best / For  $p_{T,j_c}$ ,  $\mu_0$  best / For cross section,  $2\mu_0$  best  
→  $\mu_0$  good choice with good perturbative convergence

# PDF uncertainty

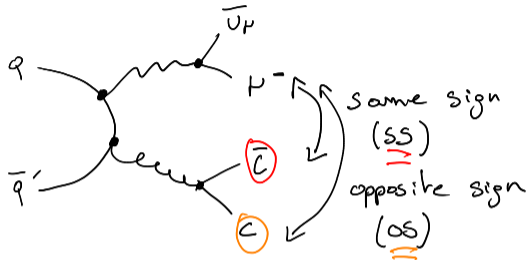
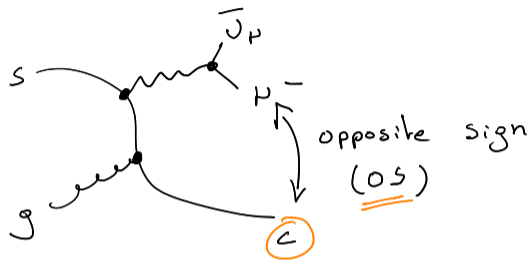


- NNPDF3.1 variation  $\sim \pm 4\%$
- Spread of various PDF sets  $\sim 10\%$ 
  - PDF error is the largest theoretical uncertainty

# Event selection(s)

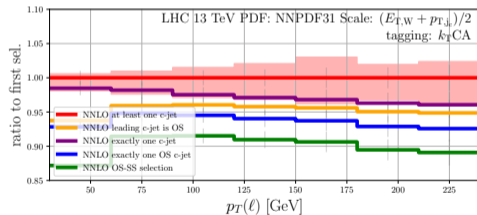
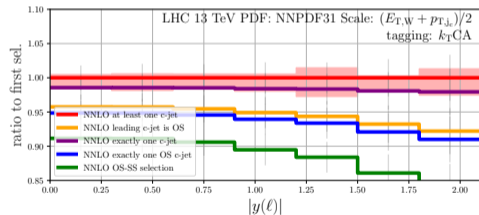
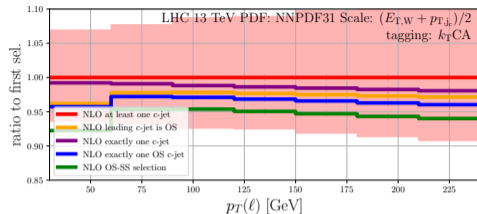
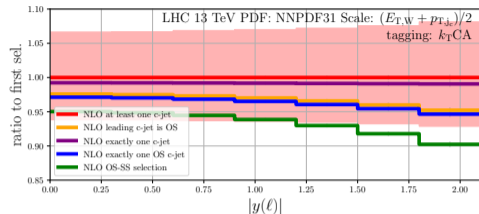


# Event selection(s)



- Experiments measure OS-SS
  - More sensitivity to strange PDF
- Many possibilities...
  - most inclusive: at least one c-jet

# Event selection(s)



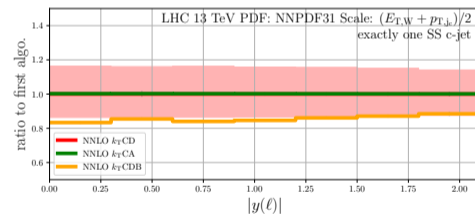
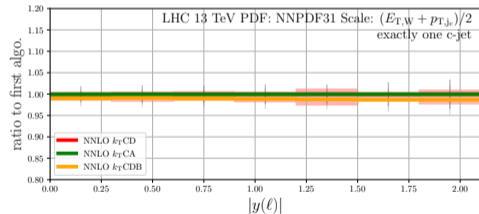
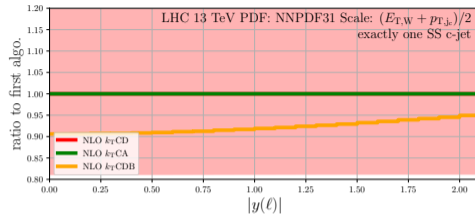
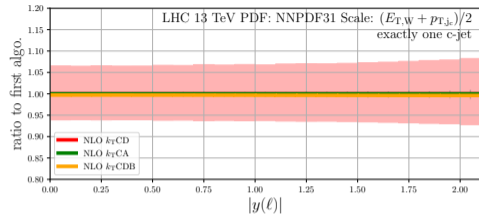
- At NLO QCD, differences covered by scale uncertainty
- At NNLO QCD, differences  $> 10\text{--}15\%$

→ Freedom in choosing whether  $cc$  and  $\bar{c}\bar{c}$  are flavoured

## Variation of flavour $k_T$ algorithm [Banfi, Salam, Zanderighi; hep-ph/0601139]

- flavoured  $k_T$  algorithm, charge dependent ( $k_T$ CD)
- flavoured  $k_T$  algorithm, charge agnostic ( $k_T$ CA)
- flavoured  $k_T$  algorithm, charge dependent, with beam definition including  $W$  momenta ( $k_T$ CDB)

# Jet algorithm (1)



- No difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet



## Jet algorithms - definitions (2)

→ Flavoured anti- $k_T$  algorithm

$$d_{ij}^{(flavored)} = d_{ij}^{(standard)} \times \begin{cases} \mathcal{S}_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavor of opposite sign,} \\ 1, & \text{otherwise.} \end{cases}$$

where

$$\mathcal{S}_{ij} = 1 - \theta (1 - \kappa_{ij}) \cos\left(\frac{\pi}{2} \kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a} \frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}.$$

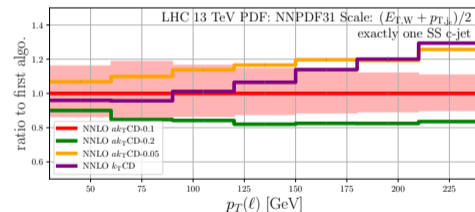
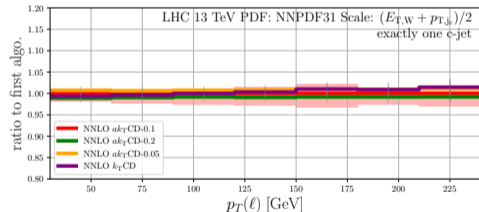
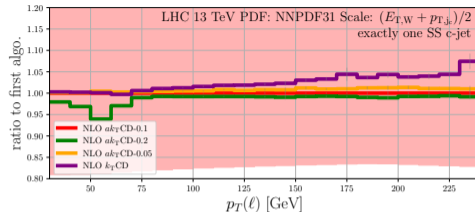
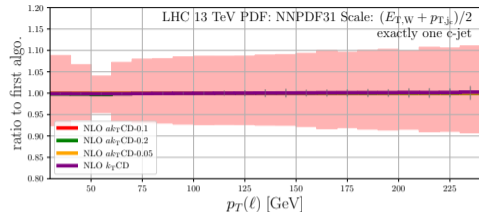
[Czakon, Poncelet, Mitov; 2205.11879]

### Variation of anti- $k_T$ algorithm

- flavoured anti- $k_T$  algorithm, charge dependent, with  $a = 0.2, 0.1, 0.05$  ( $a k_T \text{CD-}0.2, a k_T \text{CD-}0.1, a k_T \text{CD-}0.05$ )
- flavoured anti- $k_T$  algorithm, charge agnostic, with  $a = 0.1$  ( $a k_T \text{CA-}0.1$ ).

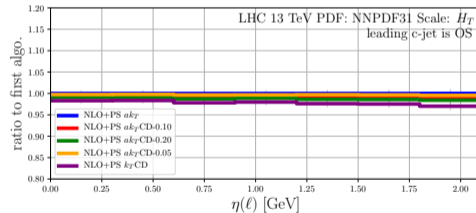
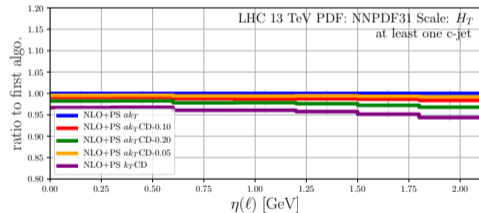
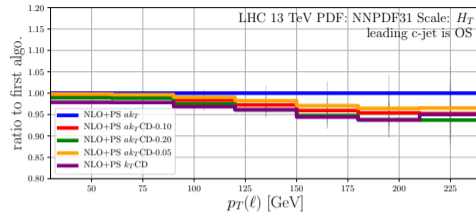
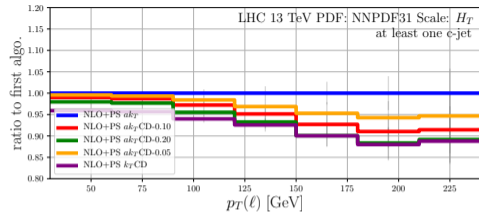
Alternatives: [Caletti, Larkoski, Marzani, Reichelt; 2205.01117, 2205.01109], [Gauld, Huss, Stagnitto; 2208.11138], [Caola et al.; 2306.07314]

# Jet algorithm (2)



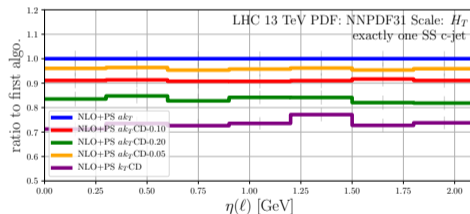
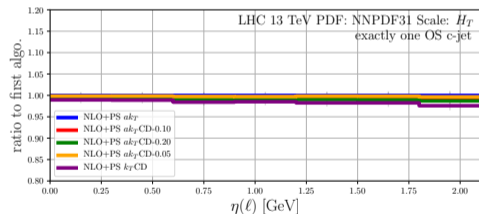
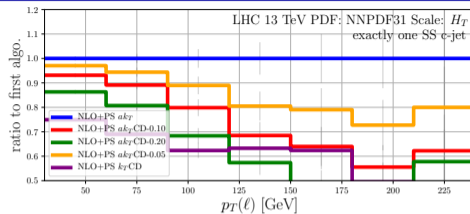
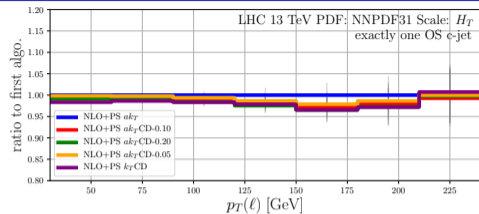
- No (small) difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

# Jet algorithm (3) at **NLO+PS**



- 5–10% differences for at least one c-jet (inclusive)
- Below 5% differences for leading c-jet is OS

# Jet algorithm (4) at **NLO+PS**



- $< 3\%$  differences for exactly one OS c-jet
- Huge differences for exactly one SS c-jet  
→ exactly one OS c-jet is preferred in this respect

- Difference in the jet algorithms (flavoured  $k_T$  vs. anti- $k_T$ )
  - ~~Estimated to be 12% in  $Z + b$  [Gauld et al., 2005.03016] ...~~
  - ... but difficult to translate to  $W + c$  ✓ → < 3% for OS selection

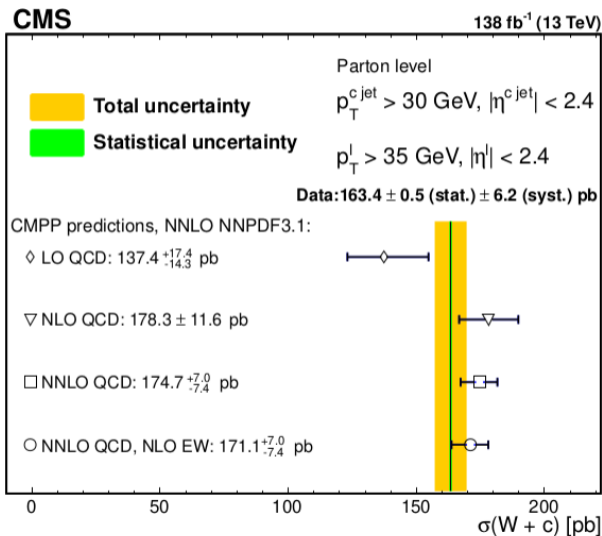
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- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
  - ~ ~~few per cent~~ ✓ **3%**

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~ ~~few per cent~~ ✓ **3%**
- Absence of EW corrections ~ ~~–few per cent~~ ✓ **–1.9%**

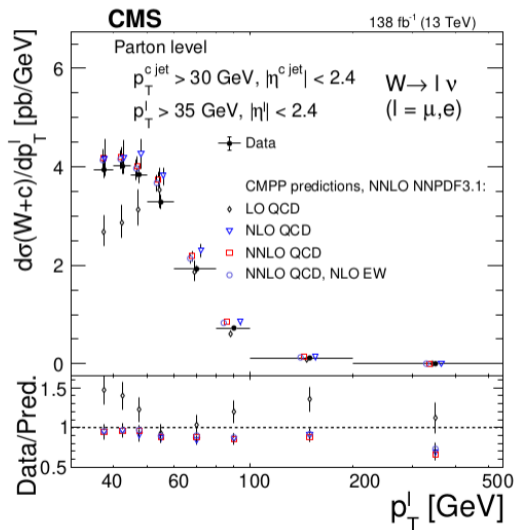
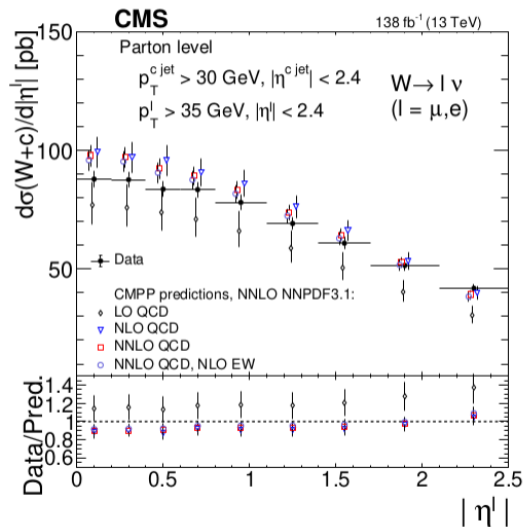
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- PDF uncertainty ~  **$\pm 4%$  or  $> 10%$  ?**

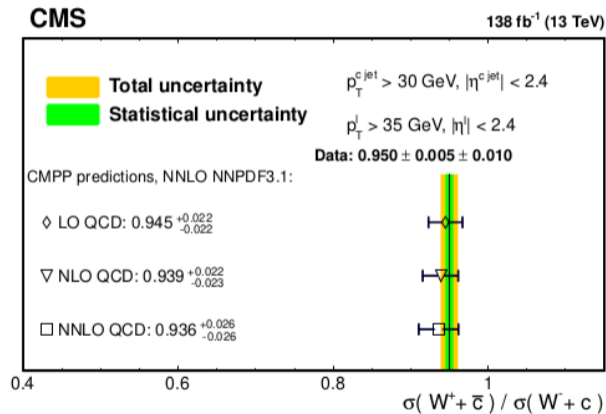


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... but difficult to translate to  $W + c$  ✓ → **< 3% for OS selection**
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element  
~ ~~few per cent~~ ✓ **3%**
- Absence of EW corrections ~ ~~few per cent~~ ✓ **-1.9%**
- PDF uncertainty ~  **$\pm 4%$  or  $> 10%$  ?**
- Definition of the experimental measurement ?  **$> 10-15%$**   
→ D meson and not charm jet ?

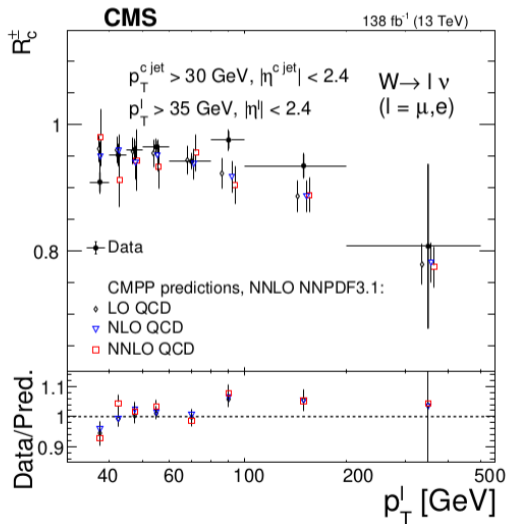
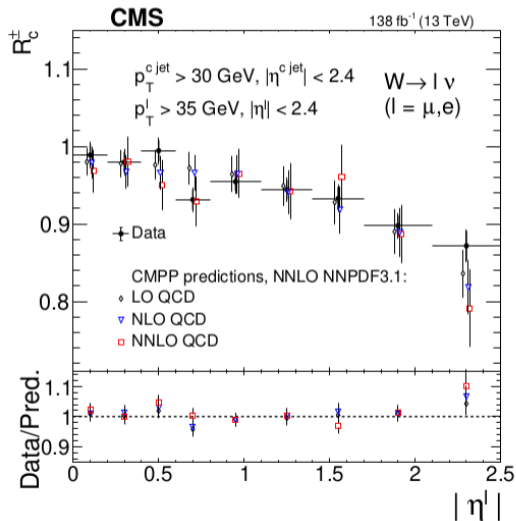


# Comparison to data [CMS + Czakon, Mitov, MP, Poncelet; 2308.02285]





# Comparison to data [CMS + Czakon, Mitov, MP, Poncelet; 2308.02285]



- $W+c$  at NNLO QCD+EW
- non-diagonal CKM effects
- PDF uncertainty
- Jet algorithm
- Experimental definition

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→ **All theoretical aspects under control apart from PDF**  
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→ **All theoretical aspects under control apart from PDF**  
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- Decisive information for SM measurements  
→ Precision programme at the LHC
- Crucial interplay between theory and experiment  
→ Big impact on physics results



# BACK-UP

QCD order	EW order	$\sigma_{W+c}^{\text{OS}}$	$\sigma_{W+c}^{\text{SS}}$	$\sigma_{W+c}^{\text{OS-SS}}$	$\Delta_{\text{stat}}^{\text{OS-SS}}$	$\Delta_{\text{scales}}^{\text{OS-SS}}$	$\Delta_{\text{PDF}}^{\text{OS-SS}}$	$\Delta_{\text{Total}}^{\text{OS-SS}}$
LO	LO	137.4	0	137.4	$\pm 0.1$	+16.6 -13.3	$\pm 5.1$	+17.4 -14.3
NLO	LO	182.4	4.1	178.3	$\pm 0.3$	+9.3 -9.4	$\pm 6.8$	+11.6 -11.6
NNLO	LO	182.9	8.2	174.7	$\pm 1.0$	+1.2 -2.8	$\pm 6.8$	+7.0 -7.4
NNLO	NLO	179.1	8.0	171.1	$\pm 1.0$	+1.2 -2.8	$\pm 6.8$	+7.0 -7.4

Based on [Czakon, Mitov, MP, Poncelet; 2212.00467]

- Charged lepton

$$p_{T,\ell} > 30 \text{ GeV}, \quad |\eta_\ell| < 2.5.$$

- At least one c-tagged jet

$$p_{T,j_c} > 20 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

$\sigma_{\text{NNLO}}$ [pb]	full CKM	$V_{cd}^{\text{LO}} \neq 0$	no CKM
+	$168.6(8)^{+0.7\% +3.8\%(\text{PDF})}_{-2.1\% -3.8\%(\text{PDF})}$	$164.4(8)^{+1.0\% +3.9\%(\text{PDF})}_{-2.4\% -3.9\%(\text{PDF})}$	$156.7(8)^{+0.7\% +4.2\%(\text{PDF})}_{-2.1\% -4.2\%(\text{PDF})}$
-	$173.9(1.9)^{+0.6\% +3.7\%(\text{PDF})}_{-1.8\% -3.7\%(\text{PDF})}$	$168.5(1.9)^{+1.0\% +3.8\%(\text{PDF})}_{-2.2\% -3.8\%(\text{PDF})}$	$156.7(1.9)^{+0.5\% +4.2\%(\text{PDF})}_{-1.6\% -4.2\%(\text{PDF})}$