

Precision in $W+c$ production at the LHC

Mathieu PELLE

University of Freiburg

Theory Seminar MPI Munich

Munich, Germany

17nd of February 2023



Outline:

→ Predictions for $W+c$! ...

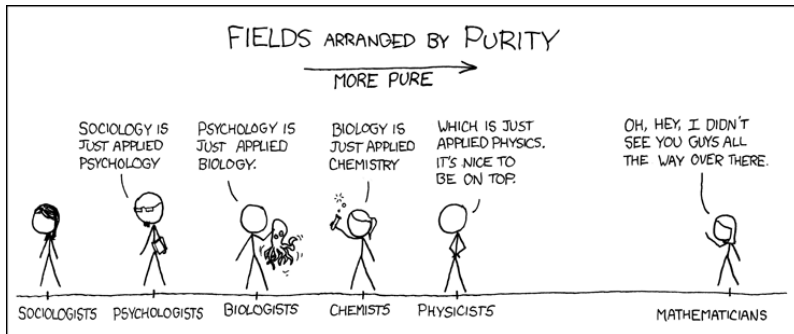
→ ... and why you want to compute them

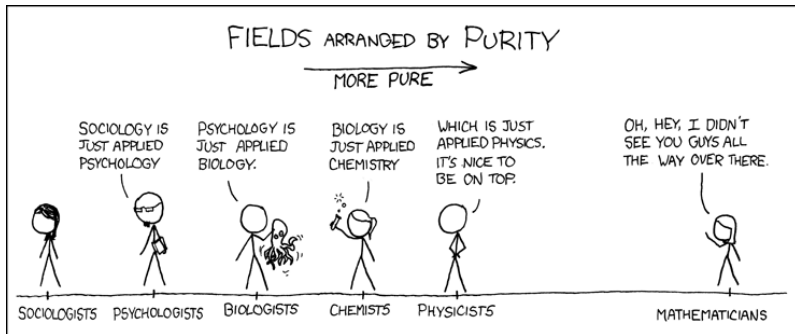
- *NNLO QCD predictions for $W+c$ -jet production at the LHC*

[Czakon, Mitov, MP, Poncelet; 2011.01011]

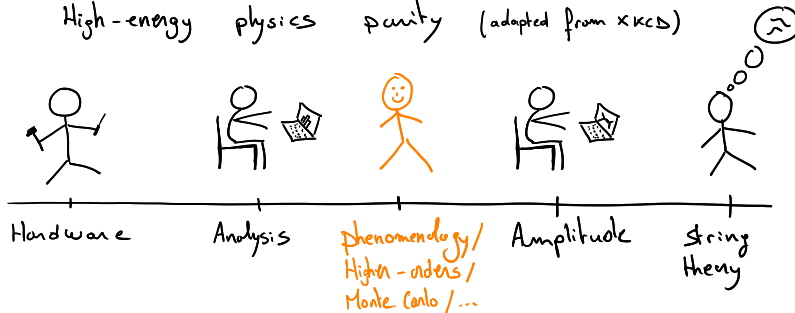
- *A detailed investigation of $W+c$ -jet at the LHC*

[Czakon, Mitov, MP, Poncelet; 2212.00467]





High-energy physics purity (adapted from XKCD)

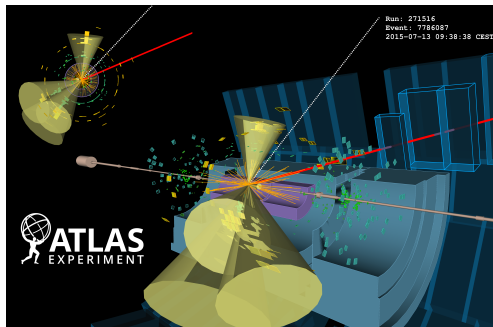




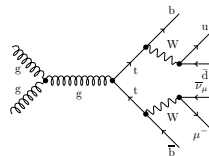
→ Illustration of Giordano Bruno's philosophical ideas (XVIth century)

LHC: Great tool to probe fundamental interactions at high energies

→ Cross talk between **experiment** and **theory**



$$pp \rightarrow t^* \bar{t}^* \rightarrow (W^* \rightarrow \nu_\mu \mu^-) (W^* \rightarrow jj) b \bar{b}$$

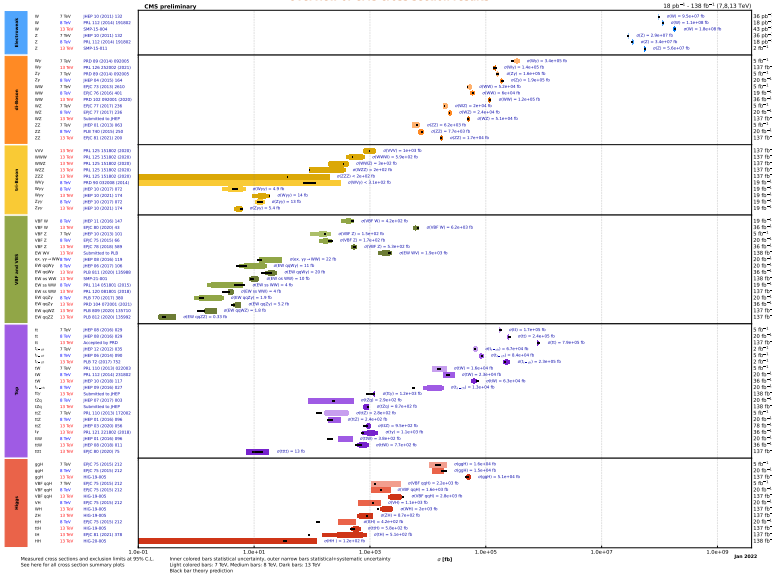


- Greatest achievement of the LHC so far:

Discovery of the Higgs boson



→ Great interest in measuring properties of the Higgs boson ...
... but there are also other interesting things



→ Cross-sections measurements machine!

- NLO QCD:

[Giele et al.; hep-ph/9302225], [Arnold et al.; Nucl.Phys. B319 (1989) 37-71, Phys.Rev. D40 (1989) 912], [Campbell et al.; hep-ph/0202176, 0809.3003, 1107.3714]

- NNLO QCD:

[Boughezal et al.; 1504.0213, 1602.06965], [Gehrmann-De Ridder et al.; 1901.11041], [MP et al.; 2204.12394]

- NLO EW:

[Kühn et al.; hep-ph/0703283, 0708.0476], [Hollik et al.; 0707.2553], [Denner et al.; 0906.1656]

- Combinations of QCD and EW corrections:

[Kallweit et al.; 1412.5157, 1511.08692], [Lindert et al.; 1511.08692], [Biederman, MP et al.; 1704.05783]

- NLO QCD for polarised W-jet:

[Bern et al.; 1103.5445] [Stirling, Vryonidou; 1204.6427]

- NNLO QCD for polarised W-jet:

[MP, Popescu, Poncelet; 2109.14336]

- NLO QCD for $W+c$ -jet:

[Giele, Keller, Laenen; hep-ph/9511449] [Stirling, Vryonidou; 1203.6781]

- NLO QCD+PS for $W+c$ -jet:

[Bevilacqua, Garzelli, Kardos, Toth; 2106.11261]

- Study of charm production in context of strange PDF:

[Lai et al.; hep-ph/0702268], [Yalkun, Dulat; 1908.00026], [Faura et al.; 2009.00014]

- NNLO QCD $Z+b$ -jet - [Gauld et al.; 2005.03016]

- NNLO QCD $W+bb$ - [Hartanto et al.; 2209.03280]

→ This work:

[Czakon, Mitov, MP, Poncelet; 2011.01011, 2212.00467]:

NNLO QCD computation for $W+c$ -jet production



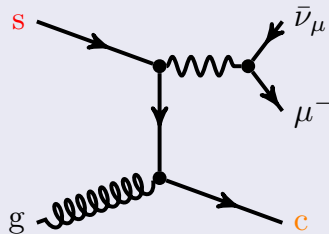
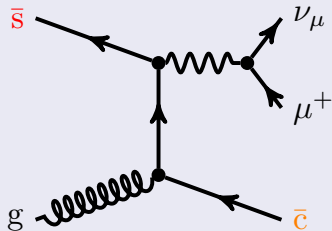
- Private Monte Carlo STRIPPER // MoCANLO
[Czakon, Heymes, Poncelet; 1005.0274, 1101.0642, 1408.2500] // [Denner, Feger, Lombardi, MP, Pelliccioli, Schmidt, Schwan]
- Tree level: AVH [Bury, van Hameren; 1503.08612]
- One-loop: OPENLOOP2 [Buccioni et al.; 1907.13071] // RECOLA [Actis et al.; 1211.6316, 1605.01090]
- Two-loop: [Gehrmann, Tancredi; 1112.1531]
→ using GINAC [Bauer, Frink, Kreckel], [Vollinga, Weinzierl; hep-ph/0410259]
- Complex-mass scheme [Denner et al.; hep-ph/9904472, hep-ph/0505042, hep-ph/0605312]
- PDF: LHAPDF [Buckley et al.; 1412.7420]

PART I

- *NNLO QCD predictions for $W+c$ -jet production at the LHC*

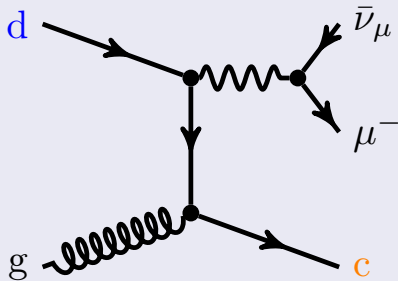
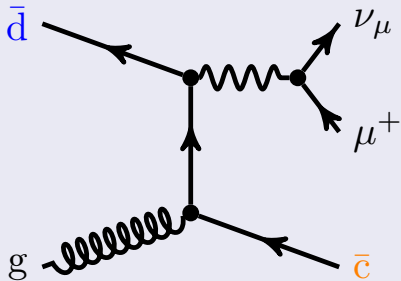
[Czakon, Mitov, MP, Poncelet; 2011.01011]

LO process (1)



- Direct link between $W+c$ measurements and strange PDF
→ main motivation to be interested in this process
- Test of (perturbative) QCD
→ s - $\bar{s}$ asymmetry predicted at 3-loop in QCD [Catani, de Florian, Rodrigo, Vogelsang; hep-ph/0404240]
- Study of flavour jets
- Further test of the Standard Model

LO process (2)



- With non-diagonal CKM matrix ($V_{cd} \neq 0$) ...
...more complicated interpretation in terms of strange PDF

LO process (3)

$$R_{W^\pm j_c} = \frac{\sigma_{W^\pm j_c}}{\sigma_{W^\pm j_c}} \sim (|V_{cs}|^2 \bar{s} + |V_{cd}|^2 \bar{d}) / (|V_{cs}|^2 s + |V_{cd}|^2 d)$$

PDF set	V_{cd}	$\sigma_{W^+ j_c}$ [pb]	$\sigma_{W^- j_c}$ [pb]	$R_{W^\pm j_c}$
NNPDF31 LO	$= 0$	9.8395(4)	10.4654(4)	0.94020(5)
	$\neq 0$	12.0725(4)	14.2624(5)	0.84646(4)
NNPDF31 NLO	$= 0$	22.593(2)	23.718(2)	0.95260(6)
	$\neq 0$	24.500(9)	27.29(1)	0.8977(5)
CT18 NLO	$= 0$	21.675(2)	21.675(2)	1.0000(1)
	$\neq 0$	23.477(9)	25.252(8)	0.9297(5)

- Large differences between PDF sets ...

Inclusion of higher orders

$$pp \rightarrow W^+ j_c$$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	✓	✓	✓
sg	X	X	✓
$s\bar{s}$	X	✓	✓
$\bar{s}s$	X	✓	✓
$\bar{s}q$	X	✓	✓
qq'	X	✓	✓
gq	X	X	✓
gg	X	✓	✓

$$pp \rightarrow W^- j_c$$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	X	X	✓
sg	✓	✓	✓
$s\bar{s}$	X	✓	✓
ss	X	✓	✓
sq	X	✓	✓
qq'	X	✓	✓
gq	X	X	✓
gg	X	✓	✓

- Higher-order corrections further complicates the picture
- Interpretation of $W+c$ -jet is not trivial

- Event selection

$$\begin{aligned} p_{T,\ell} &> 20 \text{ GeV}, & |\eta_\ell| &< 2.5 \\ p_{T,\text{miss}} &> 25 \text{ GeV}, & m_T^W &> 40 \text{ GeV}. \end{aligned}$$

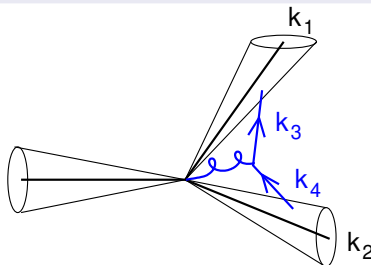
One and only one flavoured c-jet with:

$$p_{T,j_c} > 25 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

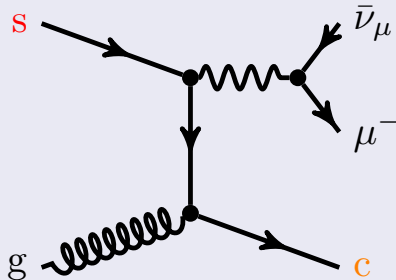
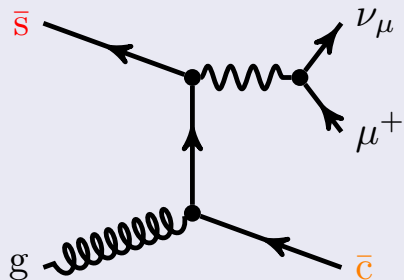
- NNPDF31 sets with $\alpha_s = 0.118$ [Ball et al.; 1706.00428]
- $\mu = \frac{1}{2} (E_{T,W} + p_{T,j_c})$ where $E_{T,W} = \sqrt{M_W^2 + (\vec{p}_{T,\ell} + \vec{p}_{T,\nu})^2}$
- flavour k_T algorithm with $R = 0.4$ [Banfi, Salam, Zanderighi; hep-ph/0601139]

Jet algorithm

- Beyond NLO, flavour jet algorithm is required
 - Otherwise not IR-safe definition of flavour jets
 - Large soft wide angle radiations are problematic
- flavour k_T algorithm with $R = 0.4$ [Banfi, Salam, Zanderighi; hep-ph/0601139]
 - Soft radiations are clustered first
 - rules:
 - $c + c = j$ or $c + \bar{c} = j$
 - $c + c + \bar{c} = j_c$ or $\bar{c} + c + \bar{c} = j_c$



Features of the computation



- NNLO QCD computation to $pp \rightarrow \mu^+ \nu_\mu j_c$ and $pp \rightarrow \mu^- \bar{\nu}_\mu j_c$
- 5-flavour scheme
- PDF uncertainty computed at NNLO using [Carrazza et al.; 1602.00005]
- $V_{cd} \neq 0$ at LO when comparing against data

Th. vs. Exp. - cross section (1)

$V_{cd} \neq 0$

Order	σ_{W+j_c} [pb]	σ_{W-j_c} [pb]	$R_{W\pm j_c} = \sigma_{W+j_c} / \sigma_{W-j_c}$
LO	$12.0725(4)^{+11.6\%}_{-12.9\%}$	$14.2624(5)^{+11.6\%}_{-10.9\%}$	$0.84646(4)^{+1.48\%}_{-2.22\%}$
NLO	$35.164(9)^{+8.0\%}_{-7.0\%}$	$37.096(9)^{+7.5\%}_{-6.7\%}$	$0.9479(3)^{+0.49\%}_{-0.36\%}$
NNLO	$38.6(1)^{+2.2\%}_{-3.2\%} \text{ } ^{+3.8\%(\text{PDF})}_{-3.8\%(\text{PDF})}$	$39.3(1)^{+1.8\%}_{-2.9\%} \text{ } ^{+3.9\%(\text{PDF})}_{-3.9\%(\text{PDF})}$	$0.983(5)^{+0.45\%}_{-0.37\%} \text{ } ^{+2.7\%(\text{PDF})}_{-2.7\%(\text{PDF})}$

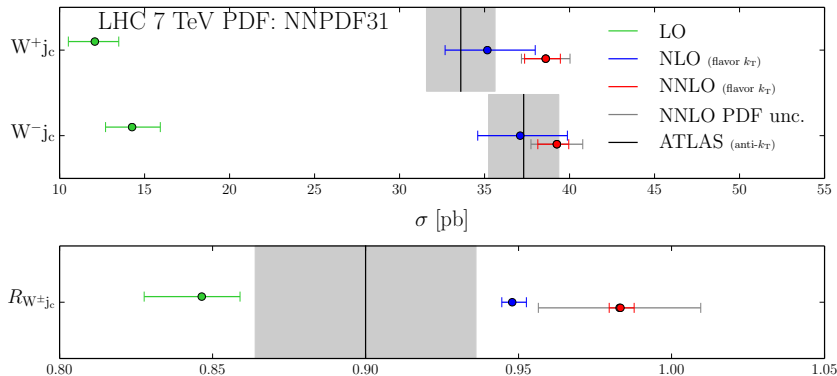
$$\sigma_{W+j_c}^{\text{ATLAS}} = 33.6 \pm 0.9 \text{ (stat)} \pm 1.8 \text{ (syst) pb}$$

$$\sigma_{W-j_c}^{\text{ATLAS}} = 37.3 \pm 0.8 \text{ (stat)} \pm 1.9 \text{ (syst) pb}$$

$$R_{W\pm j_c}^{\text{ATLAS}} = 0.90 \pm 0.03 \text{ (stat)} \pm 0.02 \text{ (syst)}$$

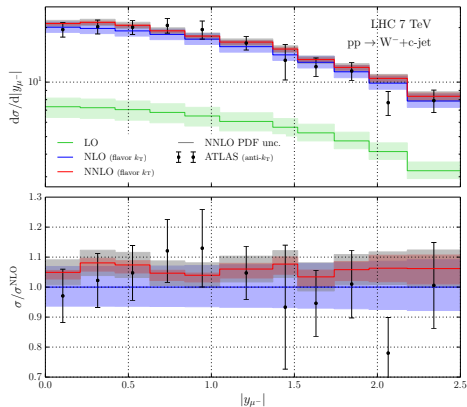
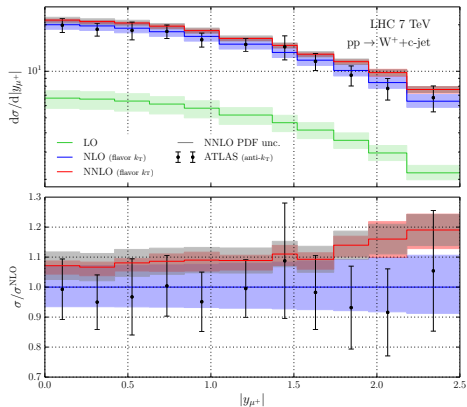
[ATLAS; 1402.6263]

Th. vs. Exp. - cross section (2)



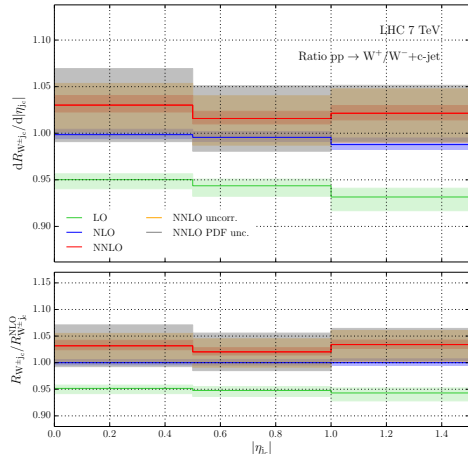
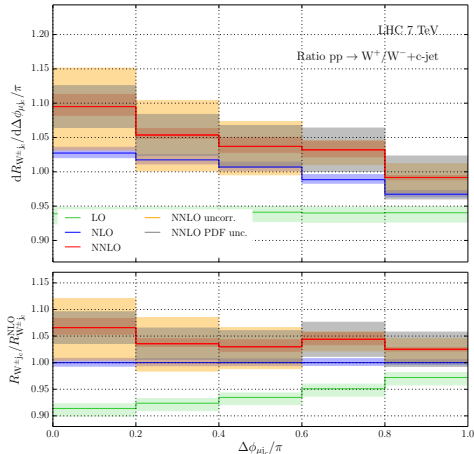
- PDF uncertainty dominant over NNLO scale uncertainty
- NNLO QCD prediction tends to be larger for the $+$ signature
- Not statistically relevant

Th. vs. Exp. - Differential distribution



Similar picture as for the total cross section
→ General good agreement

Differential distributions - ratio



As for total cross section, PDF uncertainty are dominant in ratios

→ Uncorrelated scale uncertainty more conservative

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
 - ... but difficult to translate to $W + c$

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
 - ... but difficult to translate to $W + c$
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ few per cent

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
 - ... but difficult to translate to $W + c$
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
 $\sim$ few per cent
- Absence of EW corrections $\sim$ - few per cent

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
 - ... but difficult to translate to $W + c$
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
 $\sim$ few per cent
- Absence of EW corrections $\sim$ - few per cent
- PDF uncertainty

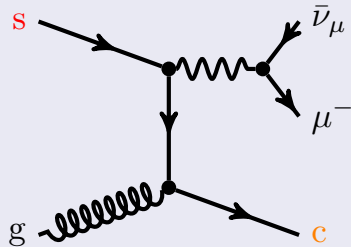
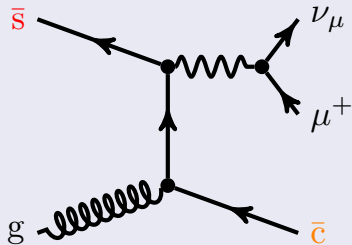
- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...
 - ... but difficult to translate to $W + c$
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element $\sim$ few per cent
- Absence of EW corrections $\sim$ - few per cent
- PDF uncertainty
- Definition of the experimental measurement ?
 - D meson and not charm jet ?

PART II

- *A detailed investigation of $W+c$ -jet at the LHC*

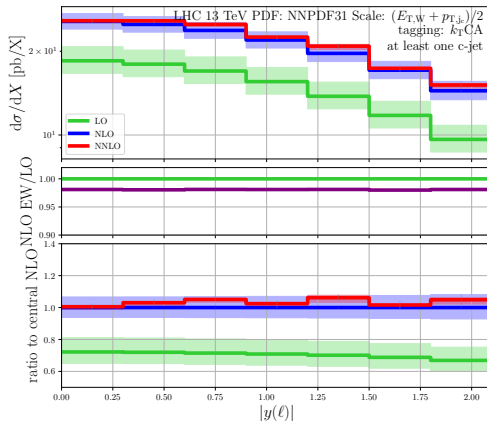
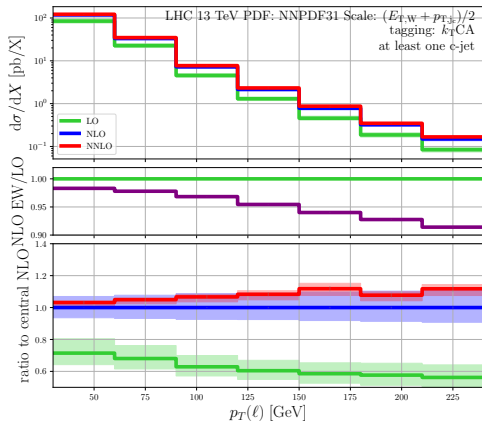
[Czakon, Mitov, MP, Poncelet; 2212.00467]

Feature of the (new) computation



- Full CKM dependence up to NNLO QCD
- NLO EW
- Study of flavour-jet algorithm
- Study of experimental definition
- 13 TeV setup

Best predictions @ 13 TeV - Differential distributions



- Good perturbative behaviour for QCD corrections
- Sudakov logarithm for EW corrections

Best predictions @ 13 TeV - cross sections

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
LO	$113.817(2)^{+12.4\%}_{-9.87\%}$	$119.711(2)^{+12.4\%}_{-9.88\%}$	$0.95076(2)^{+0.013\%}_{-0.021\%}$
NLO	$162.4(1)^{+7.2\%}_{-6.6\%}$	$168.1(1)^{+6.9\%}_{-6.4\%}$	$0.9659(9)^{+0.29\%}_{-0.21\%}$
NNLO	$168.6(8)^{+0.7\% + 3.8\%(\text{PDF})}_{-2.1\% - 3.8\%(\text{PDF})}$	$173.9(1.9)^{+0.6\% + 3.7\%(\text{PDF})}_{-1.8\% - 3.7\%(\text{PDF})}$	$0.96(1)^{+0.2\% + 2.1\%(\text{PDF})}_{-0.3\% - 2.1\%(\text{PDF})}$

- PDF uncertainty dominant at NNLO QCD

Best predictions @ 13 TeV - cross sections

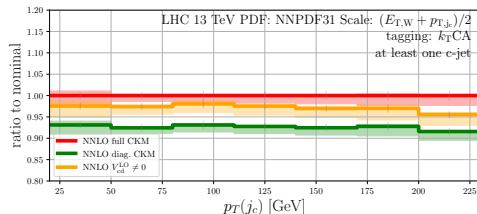
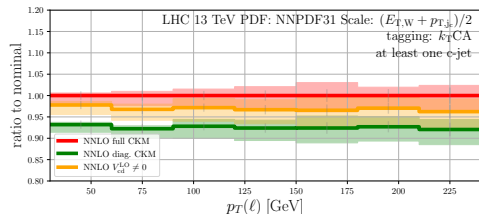
Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
LO	$113.817(2)^{+12.4\%}_{-9.87\%}$	$119.711(2)^{+12.4\%}_{-9.88\%}$	$0.95076(2)^{+0.013\%}_{-0.021\%}$
NLO	$162.4(1)^{+7.2\%}_{-6.6\%}$	$168.1(1)^{+6.9\%}_{-6.4\%}$	$0.9659(9)^{+0.29\%}_{-0.21\%}$
NNLO	$168.6(8)^{+0.7\% +3.8\%(\text{PDF})}_{-2.1\% -3.8\%(\text{PDF})}$	$173.9(1.9)^{+0.6\% +3.7\%(\text{PDF})}_{-1.8\% -3.7\%(\text{PDF})}$	$0.96(1)^{+0.2\% +2.1\%(\text{PDF})}_{-0.3\% -2.1\%(\text{PDF})}$

- PDF uncertainty dominant at NNLO QCD

Order	$\sigma_{W^+j_c}$ [pb]	$\sigma_{W^-j_c}$ [pb]	$R_{W^\pm j_c} = \sigma_{W^+j_c}/\sigma_{W^-j_c}$
NLO EW	117.399(2)	111.627(2)	0.95084(2)
$\delta_{\text{NLO EW}}[\%]$	-1.93	-1.92	-0.01

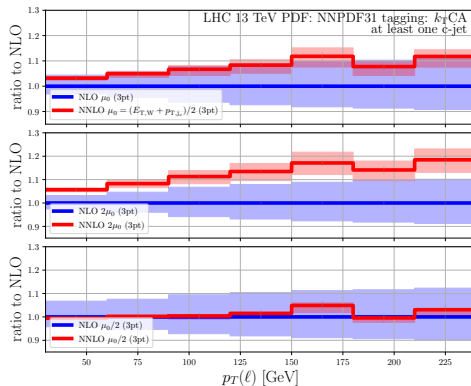
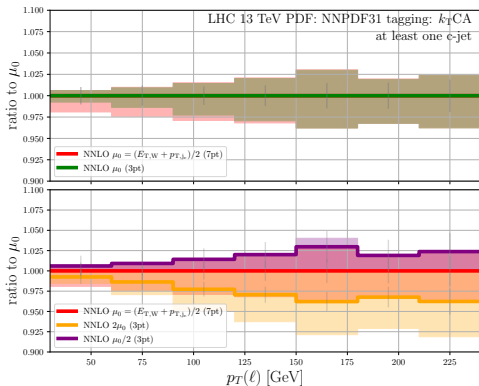
- EW corrections null in the ratio

Effect of non-diagonal CKM @ NNLO QCD



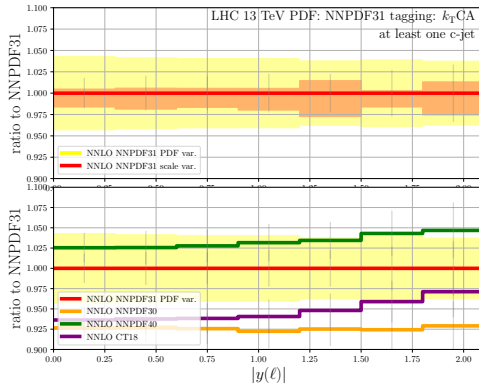
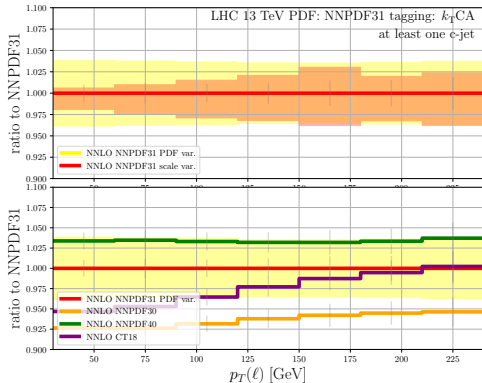
- full CKM / no CKM $\sim 7.5\text{--}11\%$
- full CKM / $V_{cd}^{LO} \neq 0 \sim 3\%$
 - Original approximation rather good
 - Full CKM dependence up to NNLO QCD for precise predictions

Scale setting



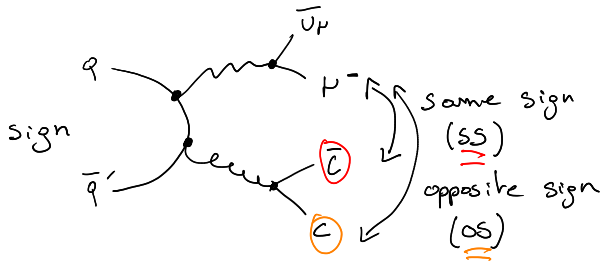
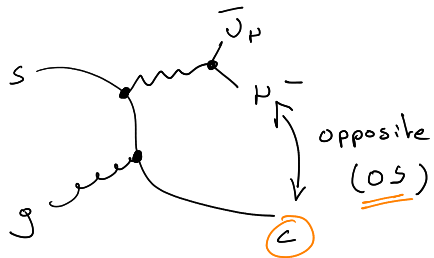
- $\mu_0 = \frac{1}{2} (E_{T,W} + p_{T,j_c})$
- For $p_{T,\ell}$, $\mu_0/2$ best / For p_{T,j_c} , μ_0 best / For cross section, $2\mu_0$ best
 → μ_0 good choice with good perturbative convergence

PDF uncertainty

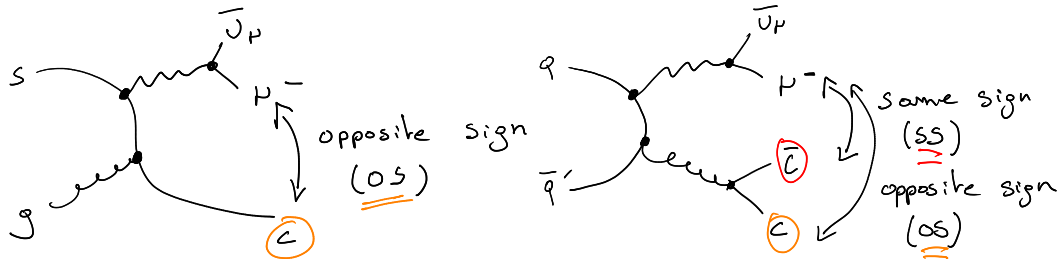


- NNPDF3.1 variation $\sim \pm 4\%$
- Spread of various PDF sets $\sim 10\%$
 - PDF error is the largest theoretical uncertainty

Event selection(s)

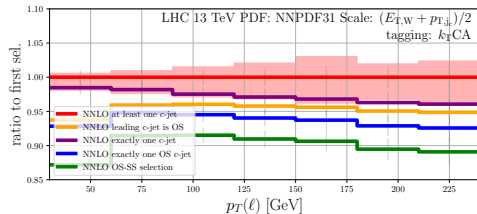
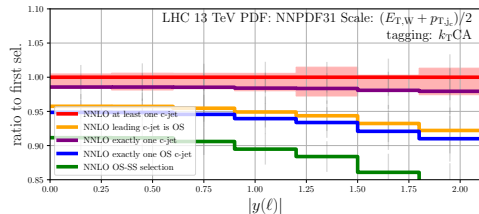
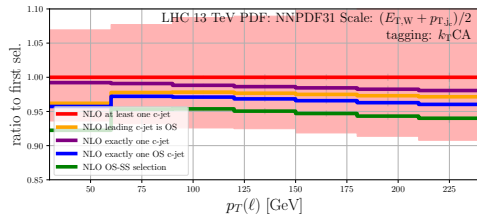
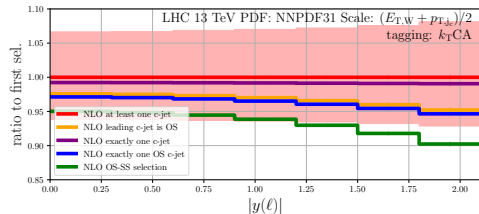


Event selection(s)



- Experiments measure OS-SS
 - More sensitivity to strange PDF
- Many possibilities...
 - most inclusive: at least one c-jet

Event selection(s)



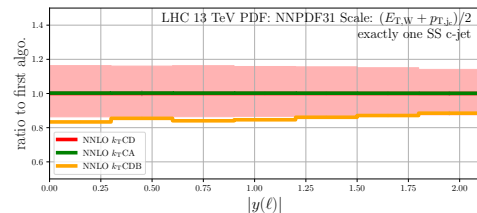
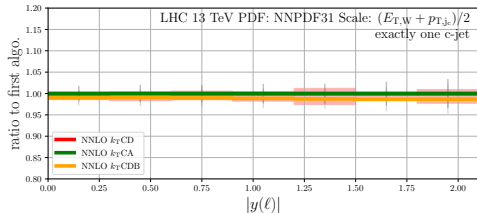
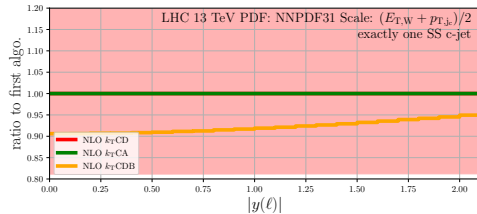
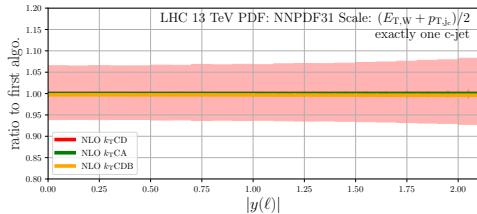
- At NLO QCD, differences covered by scale uncertainty
- At NNLO QCD, differences $> 10\text{--}15\%$

→ Freedom in choosing whether $c\bar{c}$ and $b\bar{b}$ are flavoured

Variation of flavour k_T algorithm [Banfi, Salam, Zanderighi; hep-ph/0601139]

- flavoured k_T algorithm, charge dependent (k_{TCD})
- flavoured k_T algorithm, charge agnostic (k_{TCA})
- flavoured k_T algorithm, charge dependent, with beam definition including W momenta (k_{TCDB})

Jet algorithm (1)



- No difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

Jet algorithms - definitions (2)

→ Flavoured anti- k_T algorithm

$$d_{ij}^{(flavored)} = d_{ij}^{(standard)} \times \begin{cases} \mathcal{S}_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavor of opposite sign,} \\ 1, & \text{otherwise.} \end{cases}$$

where

$$\mathcal{S}_{ij} = 1 - \theta (1 - \kappa_{ij}) \cos\left(\frac{\pi}{2} \kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a} \frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}.$$

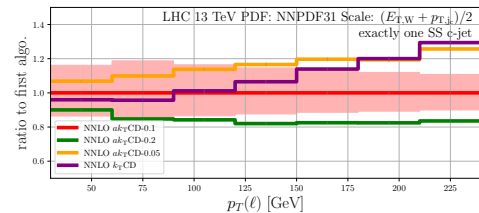
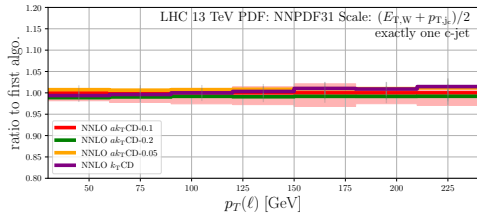
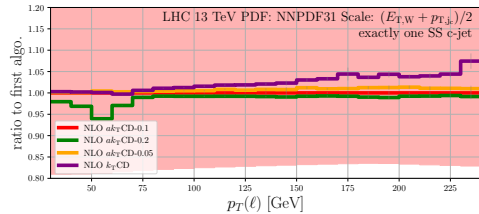
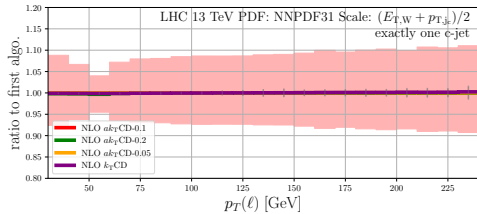
[Czakon, Poncelet, Mitov; 2205.11879]

Variation of anti- k_T algorithm

- flavoured anti- k_T algorithm, charge dependent, with $a = 0.2, 0.1, 0.05$ ($ak_T\text{CD-}0.2, ak_T\text{CD-}0.1, ak_T\text{CD-}0.05$)
- flavoured anti- k_T algorithm, charge agnostic, with $a = 0.1$ ($ak_T\text{CA-}0.1$).

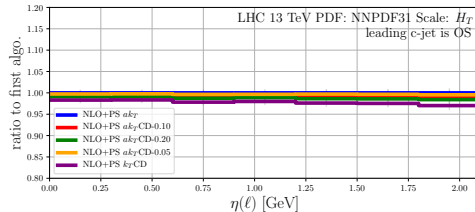
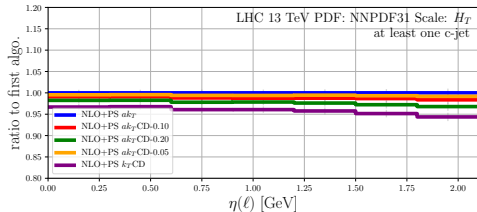
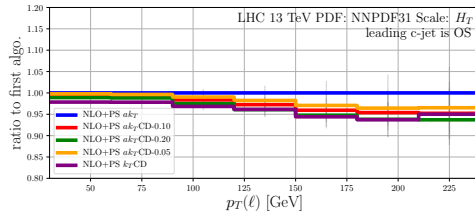
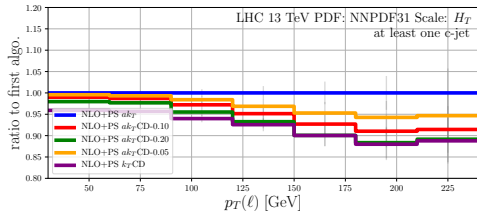
NB: Alternatives [Caletti, Larkoski, Marzani, Reichelt; 2205.01117, 2205.01109], [Gauld, Huss, Stagnitto; 2208.11138]

Jet algorithm (2)



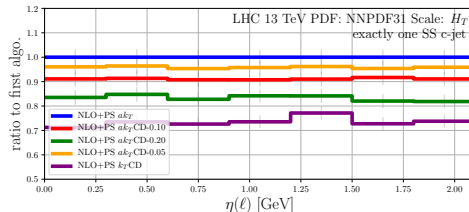
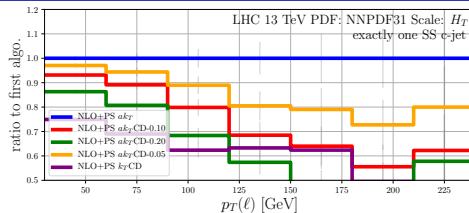
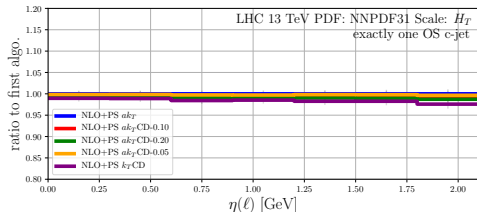
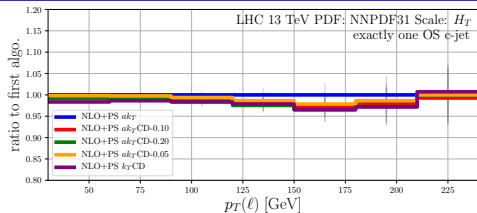
- No(small) difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

Jet algorithm (3) at **NLO+PS**



- 5–10% differences for at least one c-jet (inclusive)
- Below 5% differences for leading c-jet is OS

Jet algorithm (4) at **NLO+PS**



- $< 3\%$ differences for exactly one OS c-jet
- Huge differences for exactly one SS c-jet
→ exactly one OS c-jet is preferred in this respect

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
 - ~~Estimated to be 12% in $Z + b$ [Gauld et al., 2005.03016] ...~~
 - ... but difficult to translate to $W + c$ ✓ → **< 3% for OS selection**

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
→ ~~Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...~~
... but difficult to translate to $W + c$ ✓ → **< 3% for OS selection**
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ ~~few per cent~~ **3%**

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
→ ~~Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...~~
... but difficult to translate to $W + c$ ✓ → $< 3\%$ for OS selection
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ ~~few per cent~~ 3%
- Absence of EW corrections ~ ~~few per cent~~ -1.9%

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
→ ~~Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...~~
... but difficult to translate to $W + c$ ✓ → $< 3\%$ for OS selection
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ ~~few per cent~~ 3%
- Absence of EW corrections ~ ~~few per cent~~ -1.9%
- PDF uncertainty ~ $\pm 4\%$ or $> 10\%$?

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
→ ~~Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...~~
... but difficult to translate to $W + c$ ✓ → $< 3\%$ for OS selection
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ ~~few per cent~~ 3%
- Absence of EW corrections ~ ~~few per cent~~ -1.9%
- PDF uncertainty ~ $\pm 4\%$ or $> 10\%$?
- Definition of the experimental measurement ? $> 10-15\%$
→ D meson and not charm jet ?

Discussion reloaded

- Difference in the jet algorithms (flavoured k_T vs. anti- k_T)
→ ~~Estimated to be 12% in $Z + b$ [Gauld et al.; 2005.03016] ...~~
... but difficult to translate to $W + c$ ✓ → **< 3% for OS selection**
- Lack of higher-order QCD corrections to the off-diagonal CKM matrix element
~ ~~few per cent~~ **3%**
- Absence of EW corrections ~ ~~few per cent~~ **-1.9%**
- PDF uncertainty ~ **$\pm 4\%$ or $> 10\%$?**
- Definition of the experimental measurement ? **$> 10-15\%$**
→ D meson and not charm jet ?

Side remark: OS-SS rather artificial definition of the cross section
→ with higher orders, simple LO picture is broken

Summary - Detailed study of $W+c$

[Czakon, Mitov, MP, Poncelet; 2011.01011, 2212.00467]

- $W+c$ at NNLO QCD+EW
 - non-diagonal CKM effects
 - PDF uncertainty
 - Jet algorithm
 - Experimental definition
- All theoretical aspects under control apart from PDF

Summary - Detailed study of $W+c$

[Czakon, Mitov, MP, Poncelet; 2011.01011, 2212.00467]

- $W+c$ at NNLO QCD+EW
 - non-diagonal CKM effects
 - PDF uncertainty
 - Jet algorithm
 - Experimental definition
 - All theoretical aspects under control apart from PDF
-
- Decisive information for SM measurements
 - Precision programme at the LHC
 - Crucial interplay between theory and experiment
 - Big impact on physics results

BACK-UP

- Charged lepton

$$p_{T,\ell} > 30 \text{ GeV}, \quad |\eta_\ell| < 2.5.$$

- At least one c-tagged jet

$$p_{T,j_c} > 20 \text{ GeV}, \quad |\eta_{j_c}| < 2.5.$$

σ_{NNLO} [pb]	full CKM	$V_{cd}^{\text{LO}} \neq 0$	no CKM
+	$168.6(8)^{+0.7\% +3.8\%(\text{PDF})}_{-2.1\% -3.8\%(\text{PDF})}$	$164.4(8)^{+1.0\% +3.9\%(\text{PDF})}_{-2.4\% -3.9\%(\text{PDF})}$	$156.7(8)^{+0.7\% +4.2\%(\text{PDF})}_{-2.1\% -4.2\%(\text{PDF})}$
-	$173.9(1.9)^{+0.6\% +3.7\%(\text{PDF})}_{-1.8\% -3.7\%(\text{PDF})}$	$168.5(1.9)^{+1.0\% +3.8\%(\text{PDF})}_{-2.2\% -3.8\%(\text{PDF})}$	$156.7(1.9)^{+0.5\% +4.2\%(\text{PDF})}_{-1.6\% -4.2\%(\text{PDF})}$